15CS3109 Theory of Computation

Test - 1 : Answers

(1)

(a) Write the importance of Dead State & ϵ-Transition in FA

Ans.

Dead State: State which is not having a path to any other state in FA. Dead State appears in DFA [since every state needs exactly one Transition on each input symbol.

-> useful to complete the Transitions in DFA

ϵ-Transition: It is a transition occurs on ϵ-symbol but allowed in NFA.

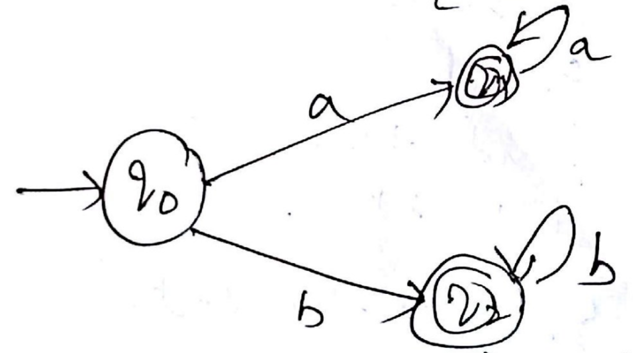
-> Allows FA to change the state without consuming an input symbol

-> Simplified FA in its design using NFA with ϵ-Transition

(b) Design NFA for the Language

$L = \{ a^n \cup b^n \} \mid n \geq 1$

Ans:



$M = \{ Q, \Sigma, \delta, q_0, F \}$ where

$Q = \{ q_0, q_1, q_2 \}$

$\Sigma = \{ a, b \}$

δ is defined above

q_0 is initial State

$\{ q_1, q_2 \}$ is set of final State.

(c) Define NFA with ϵ - Transitions. Give an example

Ans: Definition: NFA with ϵ-Transition is NFA where transitions on ϵ-Symbol are allowed along with input transitions on symbols. i.e. There are 0, 1 or more transitions on input symbols & ϵ- Symbol for each state is possible

Mathematical Representation

$M = \{Q, \Sigma, \delta, q_0, F\}$

Where Q is finite set of states

Σ is finite input alphabet

δ is transition mapping function from $Q \times \{\Sigma, \epsilon\} \rightarrow 2^Q$

q_0 is initial state.

F is set of final states $\subseteq Q$.

Example:

NFA with $L = \{0^* 1^* 2^*\}$

(or) $L = \{0^m 1^n 2^p \mid m \geq 0, n \geq 0, p \geq 0\}$

$M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1, 2\}$

δ is defined above

q_0 is initial state

$F = \{q_2\}$ is final state

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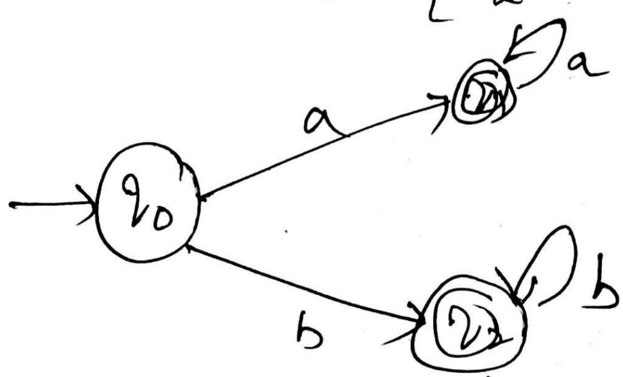

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$\Sigma = \{ a, b \}$

δ is defined above

q_0 is initial state

$\{ q_1, q_2 \}$ is set of final state.

② ⓐ Define FA & give its Mathematical representation.

Ans: FA: Finite Automata is (mathematical) machine to recongnize some set of strings derived from an input alphabet. The machine accepts an input string and gives "accept" if that string belonging to a set accepted by that FA or "Reject" if it is not.

Mathematical representation

FA is mathematically represented as 5-tuple system as $M = \{Q, \Sigma, \delta, q_0, F\}$

Where Q is finite set of states

Σ is finite set of input symbols (alphabet)

δ is a transition mapping function defined as $\delta : Q \times \Sigma \rightarrow Q$ (or) 2^Q (depending on the type of FA)

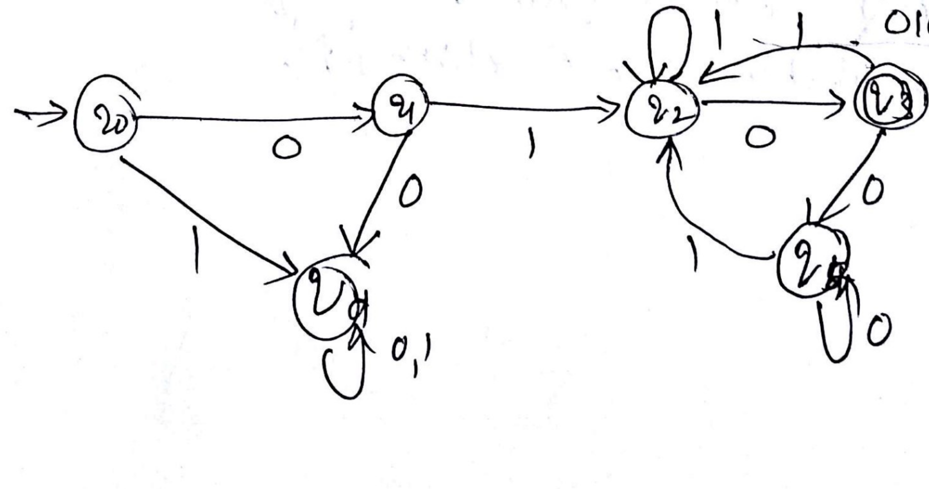
q_0 is initial state

F is set of final states $\subseteq Q$.

ⓑ Design DFA for the language $L = \{ w \mid w$ starts with '01' and ends with '10' $\}$ over $\{0,1\}$

Ans: Let the strings of the language L = { 010, 0110, 01110, 01010, 01001110, ... }

To accept the string with minimum length '010'



Let $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_d\}$

$\Sigma = \{0, 1\}$

δ is defined by diagram

q_0 is initial state

F is set of final states $= \{q_3\}$

(c) What is the importance of ϵ-closure function?

Ans: Def: ϵ-closure function

ϵ-closure$\{p\}$ = It is the set of states 'q' such that there a transition labelled on 'ε' from p to q.

→ It allows to find all the states that are having path on 'ε' in NFA from a state p.

→ If a 'q' in ε-closure{p} and if there is a ~~path~~ transition labelled on 'ε' to state 'r', then r is also in ε-closure(p)

→ It is applied in Extended Transition function ($\hat{\delta}$) to find those states that are having transitions on input symbols from Σ ~~that~~ are reachable on ε-transitions to that state. (other than ε-Transition)

Hence all the transitions are found to be runned from a state 'q' ~~are~~ if 'q' is reachable on ε-closure{p}.

③ (a) When do you say a set (or language) is regular? Justify your answer

Ans:
Def: When a language (or set) of strings is accepted by FA then that language 'L' is said to be regular.
All the languages that are accepted by FA are called Regular.

Ex: 1. L_1 = { set of strings of 0's & 1's ends with 00 }

2. L_2 = { set of strings of a's & b's that contain even no. of a's }

If a language 'L' is not accepted by 'FA' (if it is not possible to design FA), then L is not regular

Ex: $L = \{ a^m b^n \mid m > n \}$

ⓑ Explain 5-tuple Elements of Finite Automata with an example

Ans:
FA & 5-tuple Elements

$M = \{ Q, \Sigma, \delta, q_0, F \}$

Where Q is finite set of States

Σ is finite set of input symbols (alphabet)

δ is transition function

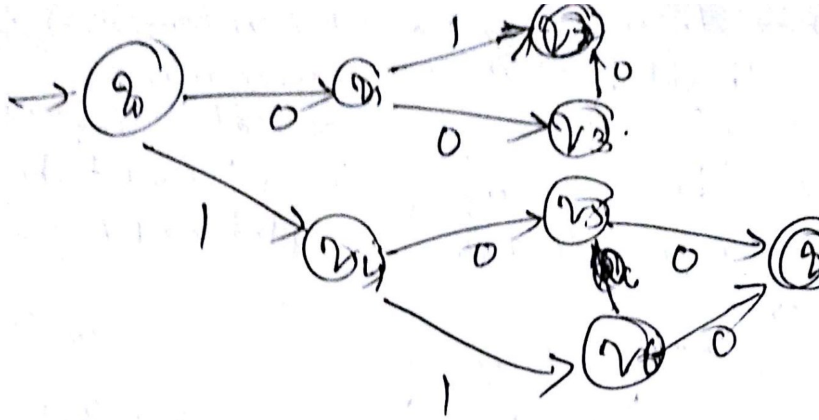
$\delta : Q \times \Sigma \rightarrow Q$ or 2^Q

q_0 is initial state

F is set of final states $\subseteq Q$

Example: The following is FA which accepts the strings either from the following

{ 01, 110, 100, 000 }



Here $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Sigma = \{0,1\}$

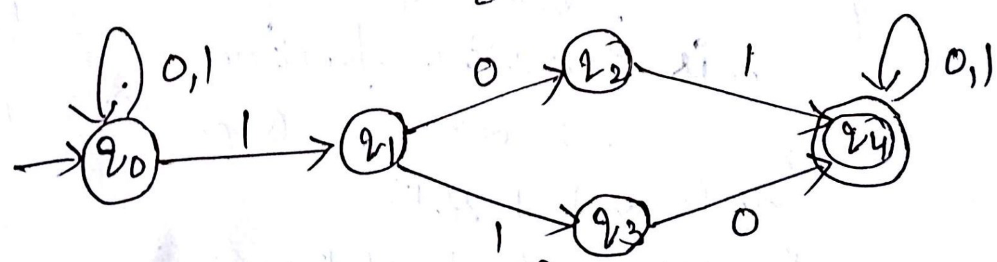
δ is defined in the diagram

q_0 is initial state

$F = \{q_2, q_7\}$ is set of final states

(3) (c) Design NFA for set of all strings such that string containing either '101' or '110' as a substring over $\{0,1\}$

Let the strings be $L = \{101, 110, 0101, 0110, \ldots\}$



$M = \{Q, \Sigma, \delta, q_0, F\}$

where $Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0,1\}$

δ is defined as above

q_0 is initial state

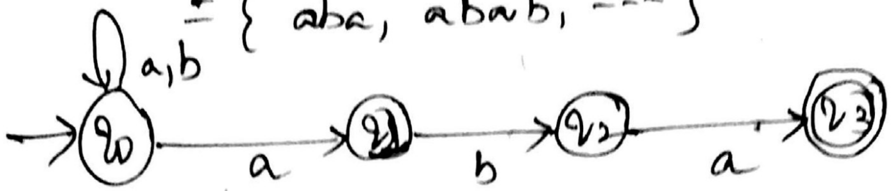
$F = \{q_4\}$ is final state

(4) (a) Design NFA to accept the set of strings over {a,b} that ends with 'aba' and find it's equivalent DFA to accept the same set

Ans:

$L = \{$ set of strings of a's b's ends with 'aba' $\}$

$= \{ aba, abab, \dots \}$



NFA is defined as $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, δ is defined with Table

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
q_2	$\{q_3\}$	ϕ
$* q_3$	ϕ	ϕ

q_0 is initial state

$\{q_3\}$ is final state

DFA using subset Method

initial state of DFA is $[q_0]$

Table is

	a	b
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_2]$
$[q_0, q_2]$	$[q_0, q_1, q_3]$	$[q_0]$
$* [q_0, q_1, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_2]$

States of DFA are renamed as

$A = [q_0]$, $B = [q_0, q_1]$

$C = [q_0, q_2]$, $D = [q_0, q_1, q_3]$

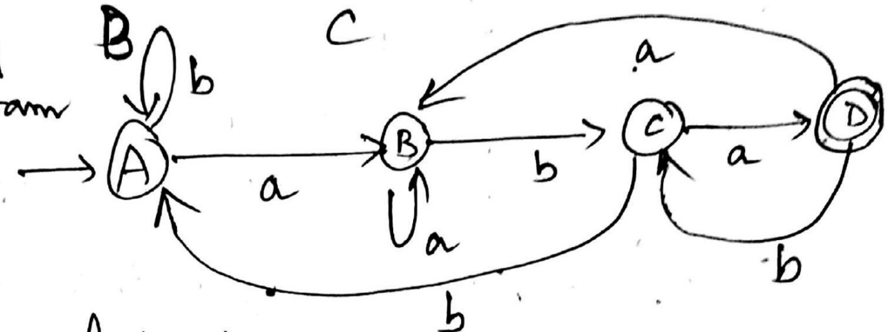
A is initial

D is final

Transition diagram is

	a	b
A	B	A
B	B	C
C	D	A
D	B	C

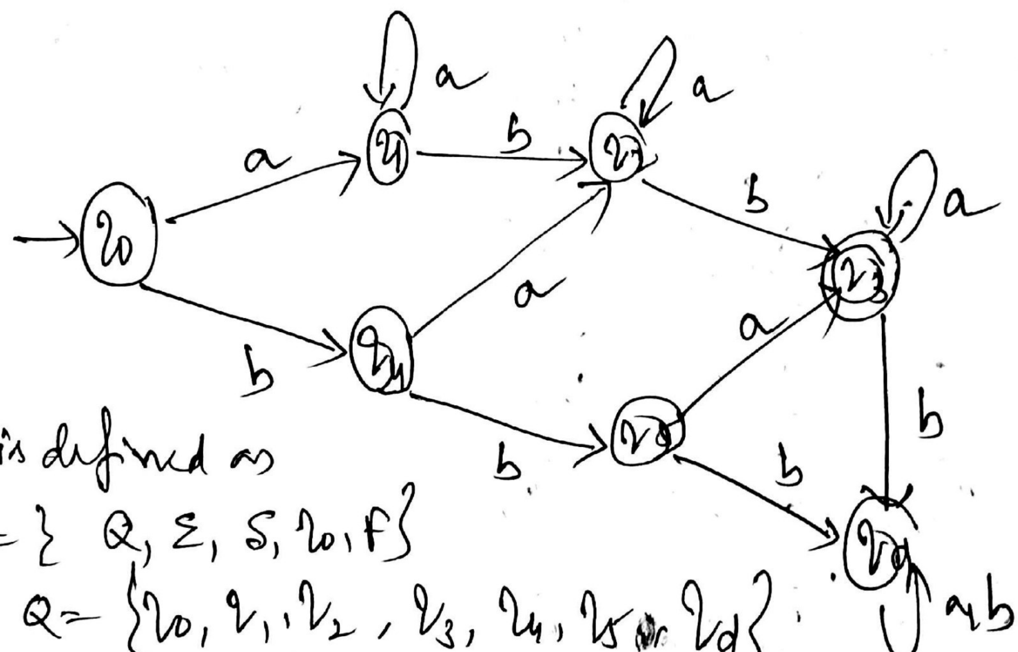
Transition Diagram



(b) Design DFA for the language $L = \{ w : n_a(w) \geq 1$ and $n_b(w) = 2$ over $\{a,b\} \}$

Ans: As per the conditions the set of strings is

$= \{ abb, bab, bba, \text{---} \}$



M is defined as

$M = \{ Q, \Sigma, \delta, q_0, F \}$

$Q = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_d \}$

$\Sigma = \{ a, b \}$

δ is defined using above diagram

q_0 is initial State

$F = \{ q_3 \}$ is set of final States

5 (a) What is the importance of ε-closure function?

Ans.

Definition: ε-closure function

ε-closure {p}: It is the set of states q such that there is a transition labelled on ε from p to q

→ It allows to find all the states that are having path labelled on ε from a state p

→ If q is in ε-closure {p} and if there is a transition labelled on ε to state r, then r is in ε-closure {p}

→ It is applied in Extended Transition function ($\hat{\delta}$) to find those states that are having transitions on input symbols from Σ (other than ε) that are reachable on ε-Transitions to that state (other than ε-Transitions) (other than ε-Transitions) (other than ε-Transitions bound to be occurred)

Hence all the transitions are bound to be reachable on ε-closure {p} from a state q if q is reachable

(b) Design Equivalent DFA for the for the given NFA where q_0 is initial state & q_3 is final state

Ans: Given NFA is

state	0	1
q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	q_2
q_2	q_3	ϕ
q_3	q_3	q_3

Let $M = \{ \{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\} \}$ is the given NFA

We define equivalent DFA as follows

$M = \{ 2^Q, \Sigma, \delta', [q_0], F' \}$ with

transition table (δ') defined as follows

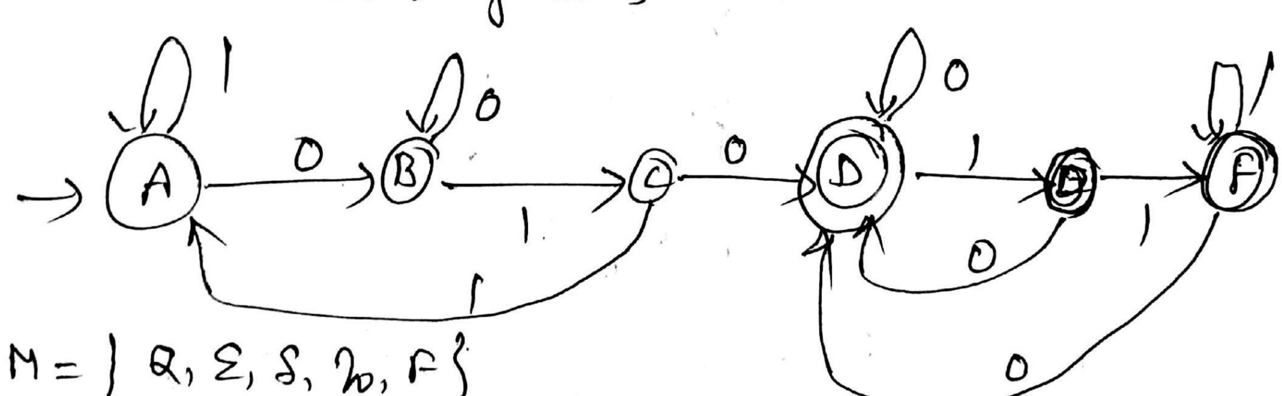
	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_2]$
$[q_0, q_2]$	$[q_0, q_1, q_3]$	$[q_0]$
$* [q_0, q_1, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_2, q_3]$
$* [q_0, q_2, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_3]$
$* [q_0, q_3]$	$[q_0, q_1, q_3]$	$[q_0, q_3]$

if we rename as $A = [q_0]$, $B = [q_0, q_1]$, $C = [q_0, q_2]$

$D = [q_0, q_1, q_3]$, $E = [q_0, q_2, q_3]$

$F = [q_0, q_3]$.

Now the transition diagram is



$M = \{ Q, \Sigma, \delta, q_0, F \}$

$Q = \{ A, B, C, D, E, F \}$, $\Sigma = \{0, 1\}$, δ is defined above

A is initial state & $F = \{ D, E, F \}$ is the set of final states

(6) (a) Specify some applications of FA

Ans: → Translators
→ compiler
→ pattern search
→ Languages & their Syntax ~~forms~~

(B) State & Prove the equivalence of NFA & DFA

Proof: Statement 1: If L is accepted by NFA then there exists DFA to accept 'L'

Statement 2: If L is accepted by DFA. Then there exists NFA to accept 'L'

Statement 1: Let NFA 'M' is defined as

$M = \{Q, \Sigma, \delta, q_0, F\}$

Where Q is set of states, Σ is input alphabet, δ is transition functions q_0 is initial & F is set of final states

We define DFA $M' = \{Q', \Sigma, \delta', q_0', F'\}$ as follows

$Q' = 2^Q$, Σ is same, δ' is defined as

$$\delta'\{[q_1, q_2 \cdots q_i], a\} = [p_1, p_2 \cdots p_j]$$

iff $$\delta\{\{q_1, q_2, \cdots q_i\}, a\} = \{p_1, p_2, \cdots p_j\}$$

i.e. $$\bigcup_{k=0}^{i} (q_i, a) = \{p_1, p_2 \cdots p_j\}$$

We prove the theorem by using Mathematical induction

Basis Let $x = \epsilon$ i.e $|x| = 0$

Then by definition of q_0' $q_0' = [q_0]$

i.e. $$\delta'([q_0], \epsilon) = \delta(q_0, \epsilon) = \{q_0\}$$

Hence it is proved for $x = \epsilon$

We assume the theorem is correct for length 'm'

i.e. $x = w$ & $|x| = m$

i.e. $$\delta'\{[q_1, q_2, \cdots q_i], x\} = [p_1, p_2, \cdots p_j]$$

iff $$\delta\{\{q_1, q_2, \cdots q_i\}, x\} = \{p_1, p_2 \cdots p_j\}$$

Scanned by CamScanner

then $\quad \delta'([q_0], xa) = \delta'\{\delta\{[q_0], x\}, a\}$

But by hypothesis $\quad \delta\{[q_1, q_2, \dots q_i], x\} = [p_1, p_2, \dots p_j]$

iff $\quad \delta\{\{q_1, q_2, \dots q_i$

& By the definition of δ' we defined

$$\delta'([q_1, q_2 \dots q_i], a) = [r_1, r_2 \dots r_s]$$

iff $\quad \delta\{\{q_1, q_2 \dots q_i\}, a\} = \{r_1, r_2 \dots r_s]$

Hence it is proved for the length $m+1$.

By the definition of F', F' contains those states which are with atleast one final state final states of NFA

Hence it is proved with acceptance that there is DFA that accepts L

Statement 2:

By the definition of NFA, every DFA is NFA, Hence second statement is correct Hence DFA $\rightarrow$ NFA

$\therefore$ NFA $\rightleftarrows$ DFA, is proved with Statement 1 & Statement 2

Ⓒ Give NFA with $M = \{ [q_0, q_1, q_2, q_3, q_4], \{0, 1\}$ δ is defined with table, q_0 is initial state, q_2 & q_4 are final state

δ is defined

State	0	1
q_0	$\{q_0, q_3\}$	$[q_0, q_1\}$
q_1	ϕ	$[q_2\}$
q_2	q_2	q_2
q_3	q_4	ϕ
q_4	$\{q_4\}$	$\{q_4\}$

Give DFA is δ'

	0	1
$[q_0]$	$[q_0, q_3]$	$[q_0, q_1]$
$[q_0, q_3]$	$[q_0, q_3, q_4]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_3]$	$[q_0, q_1, q_2]$
$[q_0, q_3, q_4]$	$[q_0, q_3, q_4]$	$[q_0, q_1, q_4]$
$[q_0, q_1, q_2]$	$[q_0, q_3, q_3]$	$[q_0, q_1, q_2]$
$[q_0, q_1, q_4]$	$(q_0, [illegible], q_3, q_4)$	$[q_0, q_1, q_2, q_4]$